Quantum Information with Solid-State Devices

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Dr. Johannes Majer

Lecture 9

RF-SQUID

Quantum superposition of distinct macroscopic states

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external flux Φ_x applied to the loop. The dynamics of the SQUID can be described in terms of the variable Φ and are analogous to those of a particle of 'mass' C (and kinetic energy $\frac{1}{2}C\dot{\Phi}^2$) moving in a one-dimensional potential (Fig. 1a) given by the sum of the magnetic energy of the loop and the Josephson coupling energy of the junction:

$$U = U_0\left[\frac{1}{2}\left(\frac{2\pi(\Phi-\Phi_x)}{\Phi_0}\right)^2 - \beta_L\cos(2\pi\Phi/\Phi_0)\right] \qquad (1)$$

where Φ_0 is the flux quantum, $U_0 \equiv \Phi_0^2/4\pi^2 L$ and $\beta_L \equiv 2\pi L I_c/\Phi_0$. For the parameters used in our experiment, this a double-well potential separated by a barrier with a height depending on I_c. When $\Phi_x = \Phi_0/2$ the potential is symmetric. Any change in Φ_x then tilts the potential, as shown in Fig. 1a.

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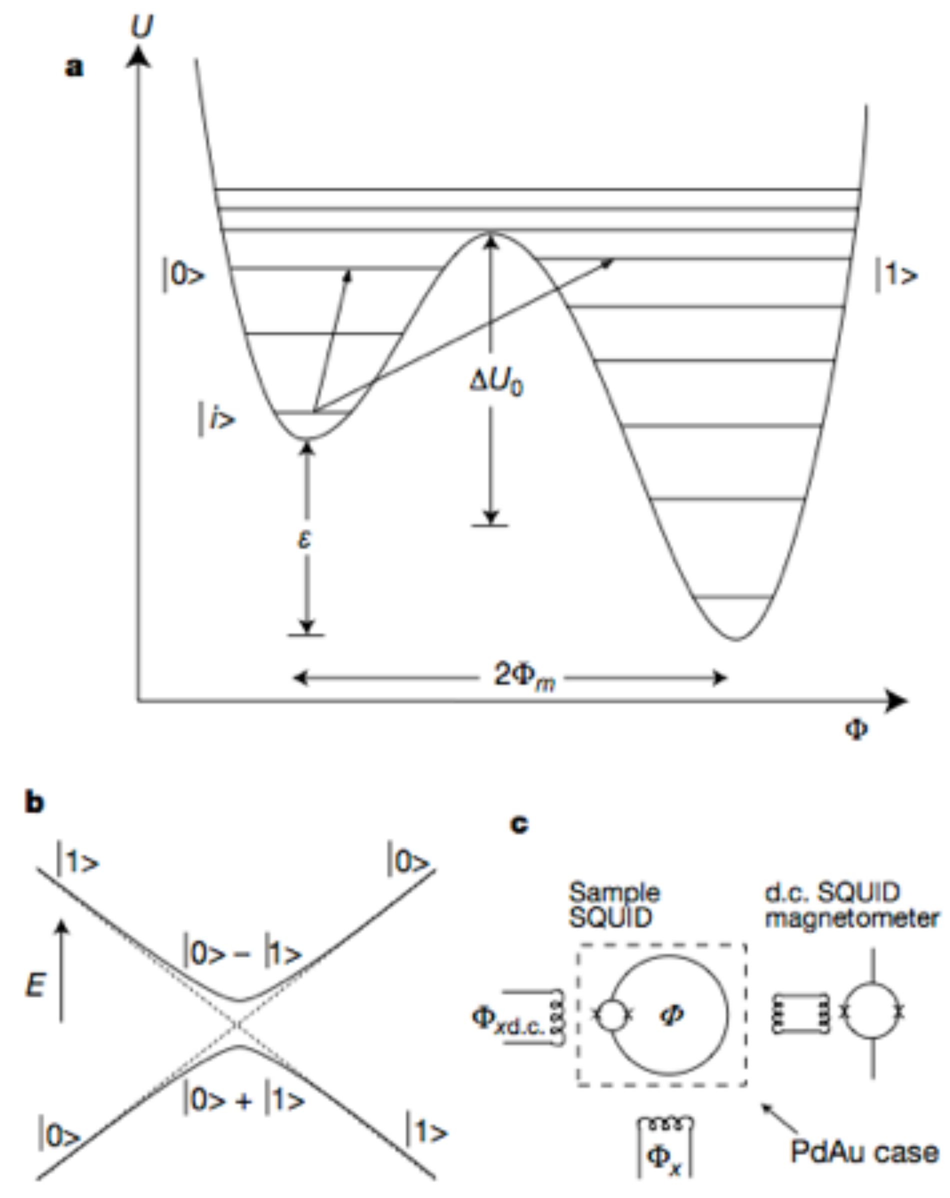
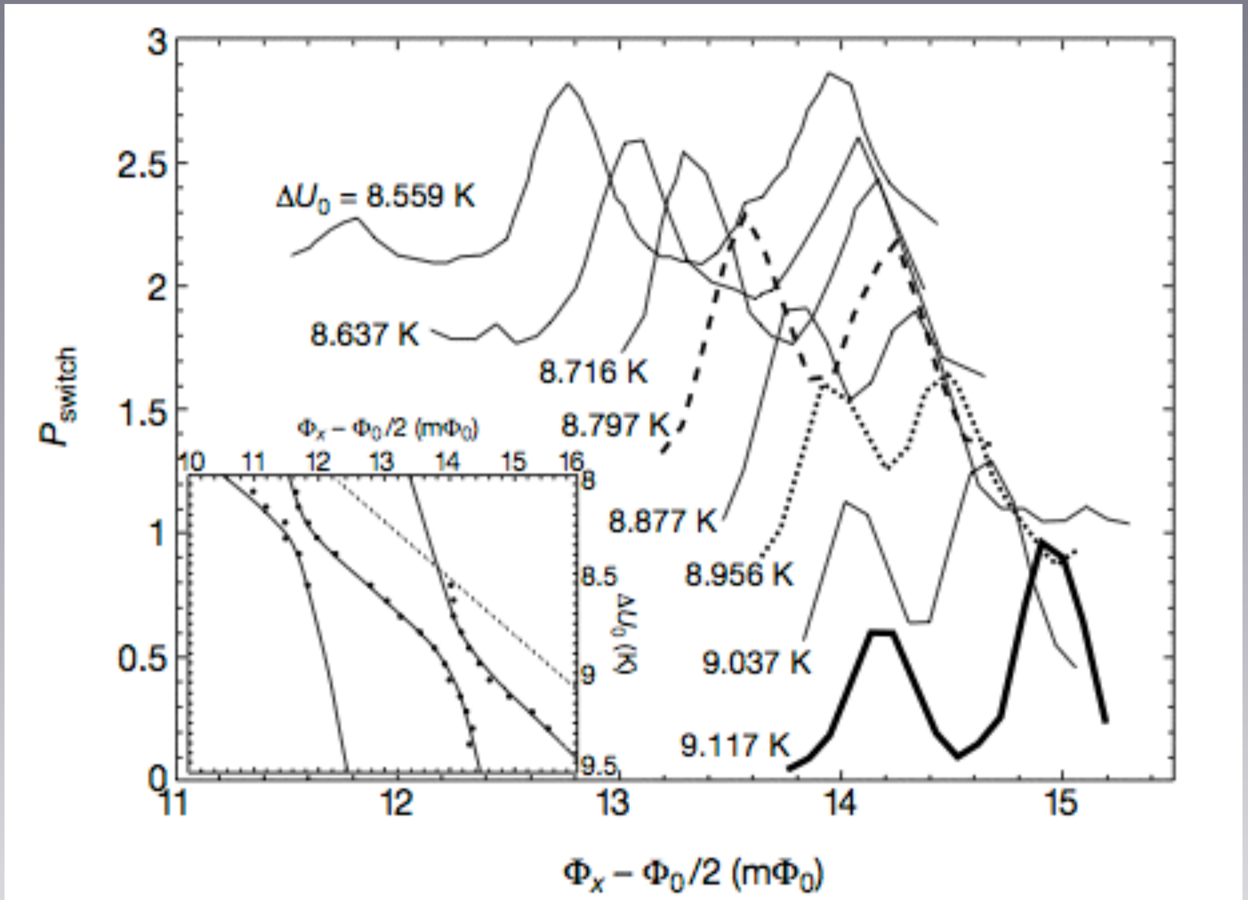
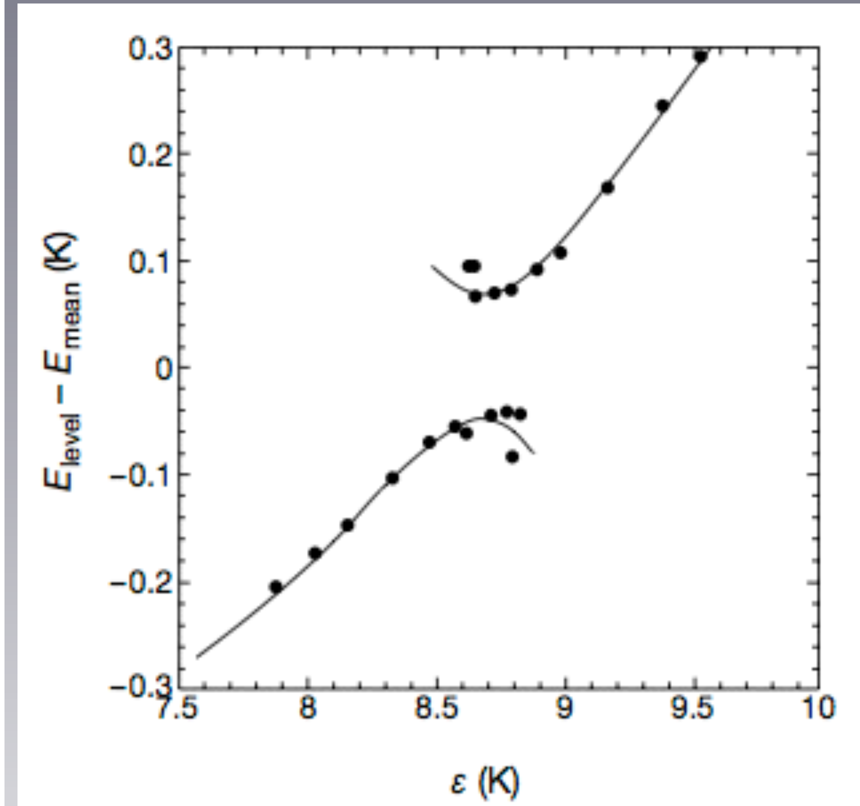


RF-SQUID



ΔU_0 = 8.559 K
8.637 K
8.716 K
8.797 K
8.877 K
8.956 K
9.037 K
9.117 K
P_{switch}
$\Phi_x - \Phi_0/2$ (mΦ_0)
ΔU_0 (K)



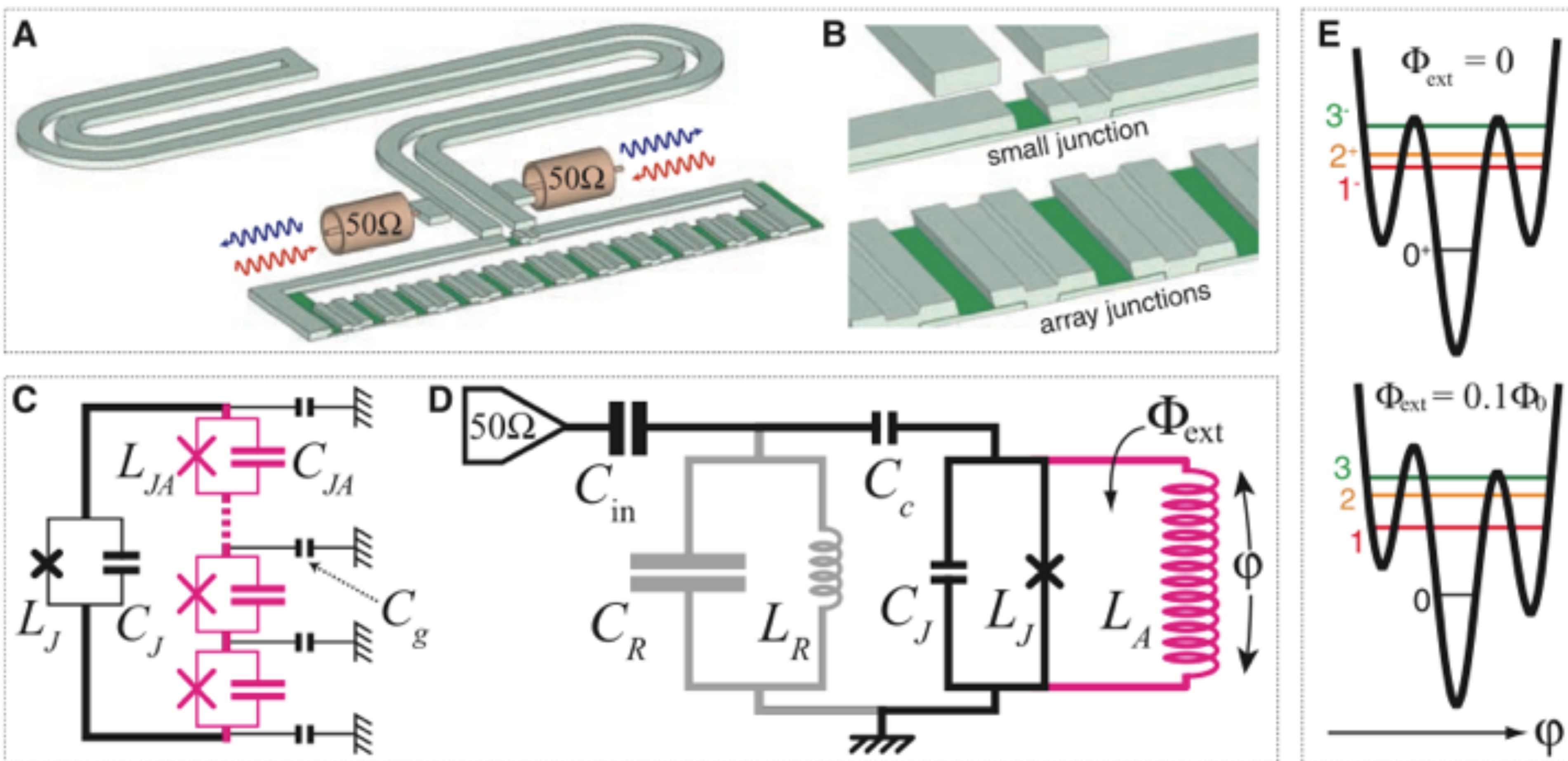
$E_{level} - E_{mean}$ (K)
ε (K)

Fluxonium

www.sciencemag.org SCIENCE VOL 326 2 OCTOBER 2009

Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets

Vladimir E. Manucharyan, Jens Koch, Leonid I. Glazman, Michel H. Devoret*



Phase Qubit

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Energy-Level Quantization in the Zero-Voltage State of a Current-Biased Josephson Junction

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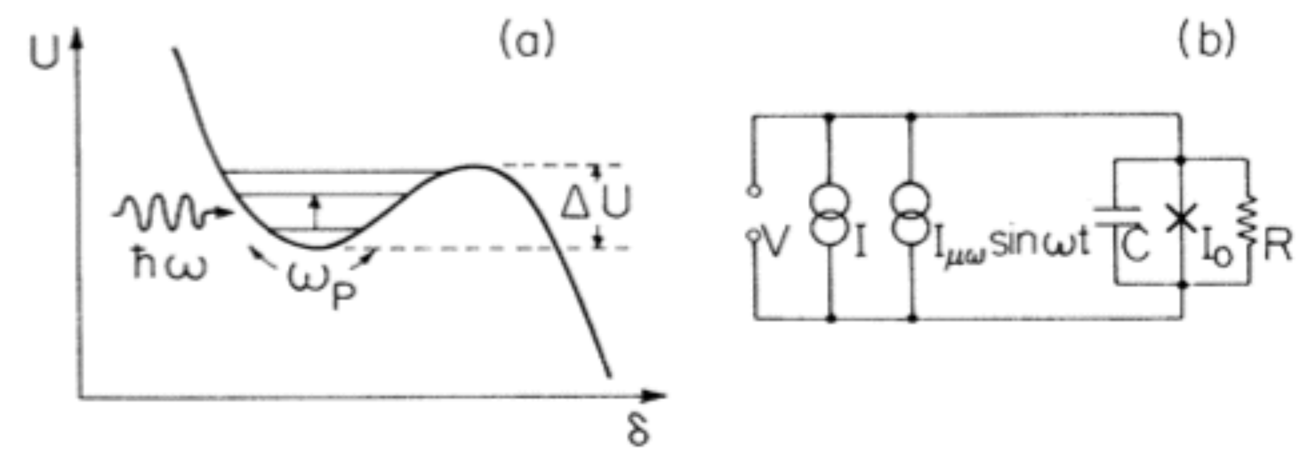
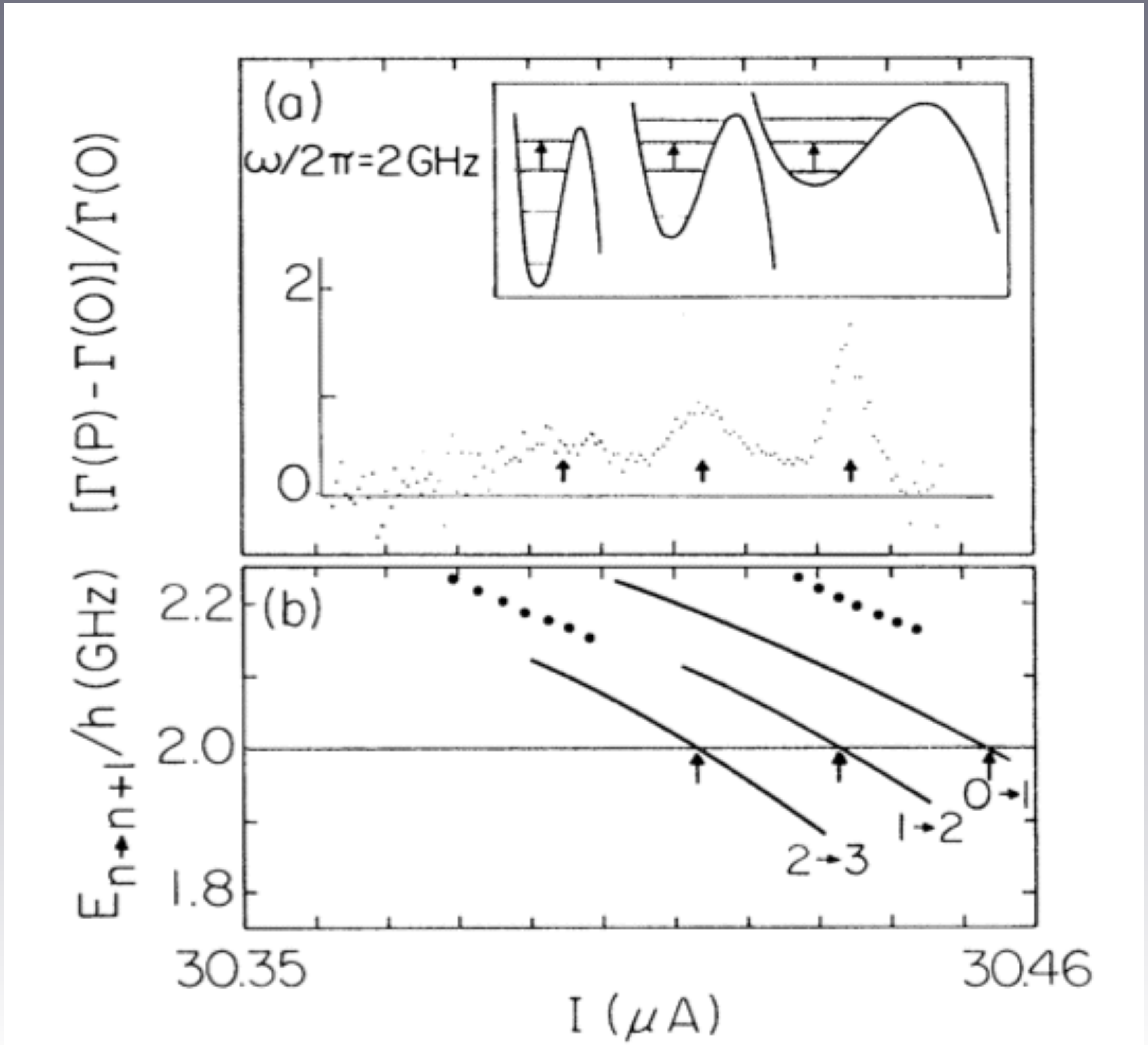
(Received 14 June 1985)

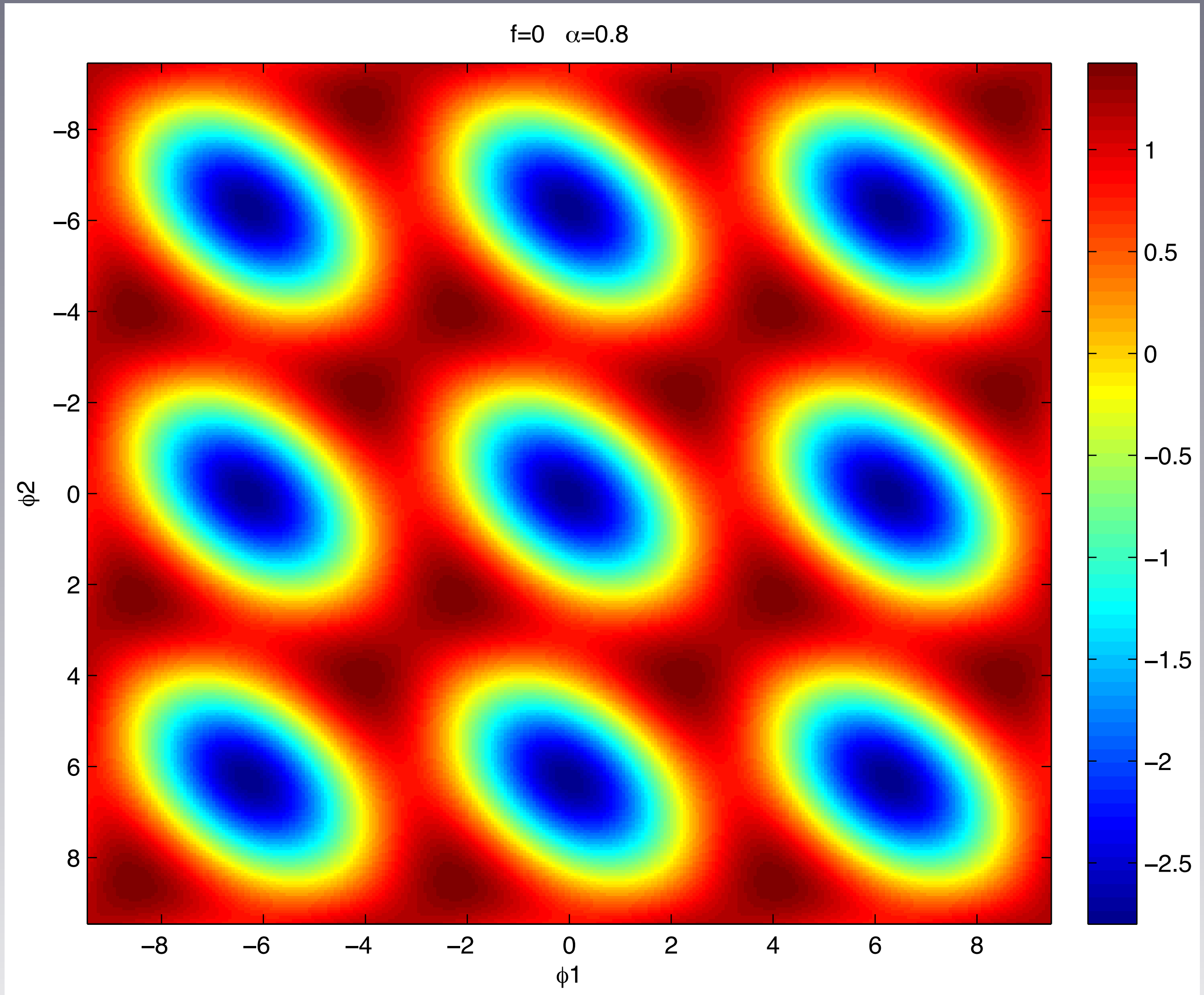


Phase Qubit



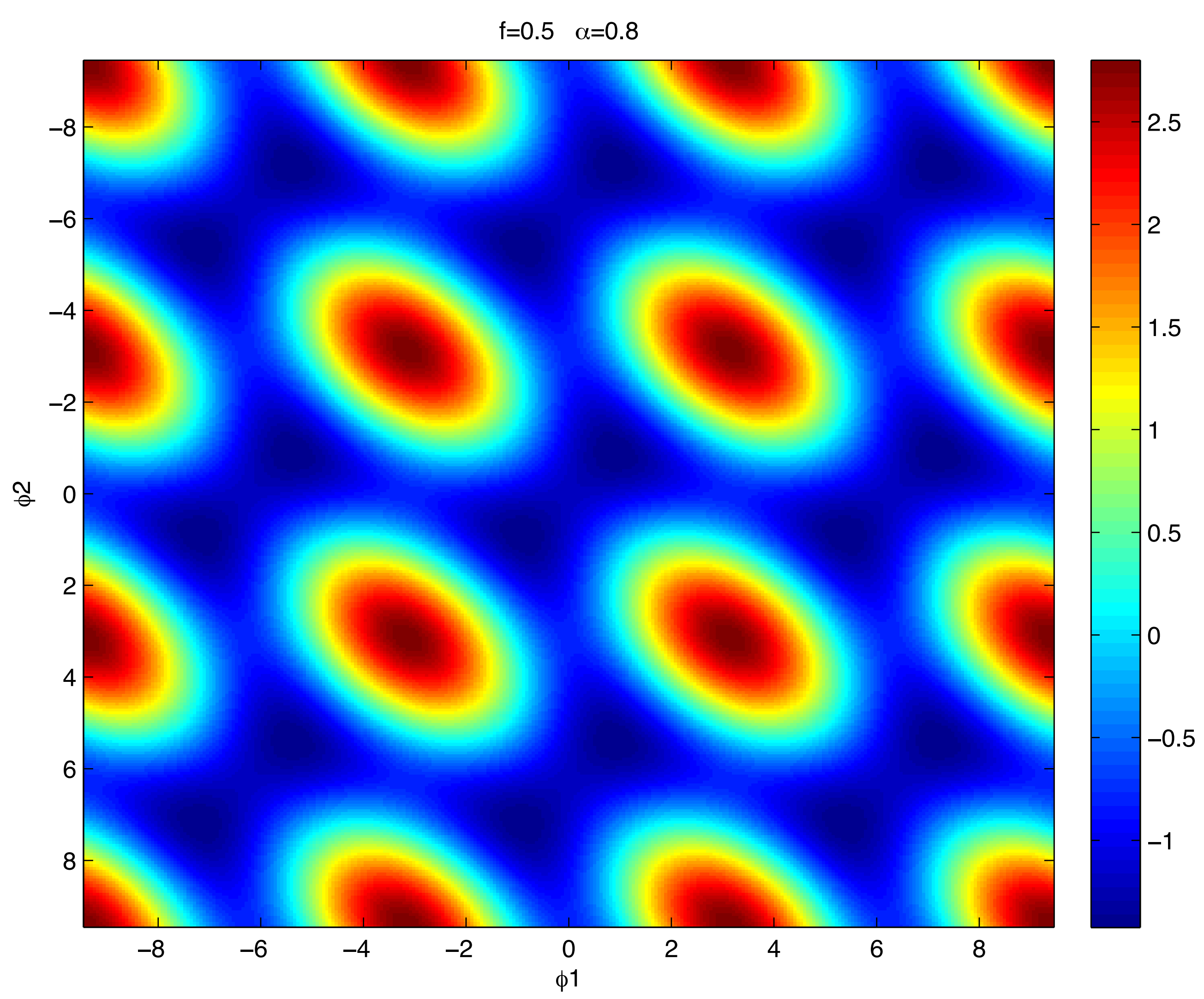
(a)
ω/2π=2GHz
[Γ(P)-Γ(O)]/Γ(O)
2
0
(b)
$E_{n \to n+1}/h$ (GHz)
2.2
2.0
1.8
0→1
1→2
2→3
30.35
30.46
I (μA)

persistent-current qubit



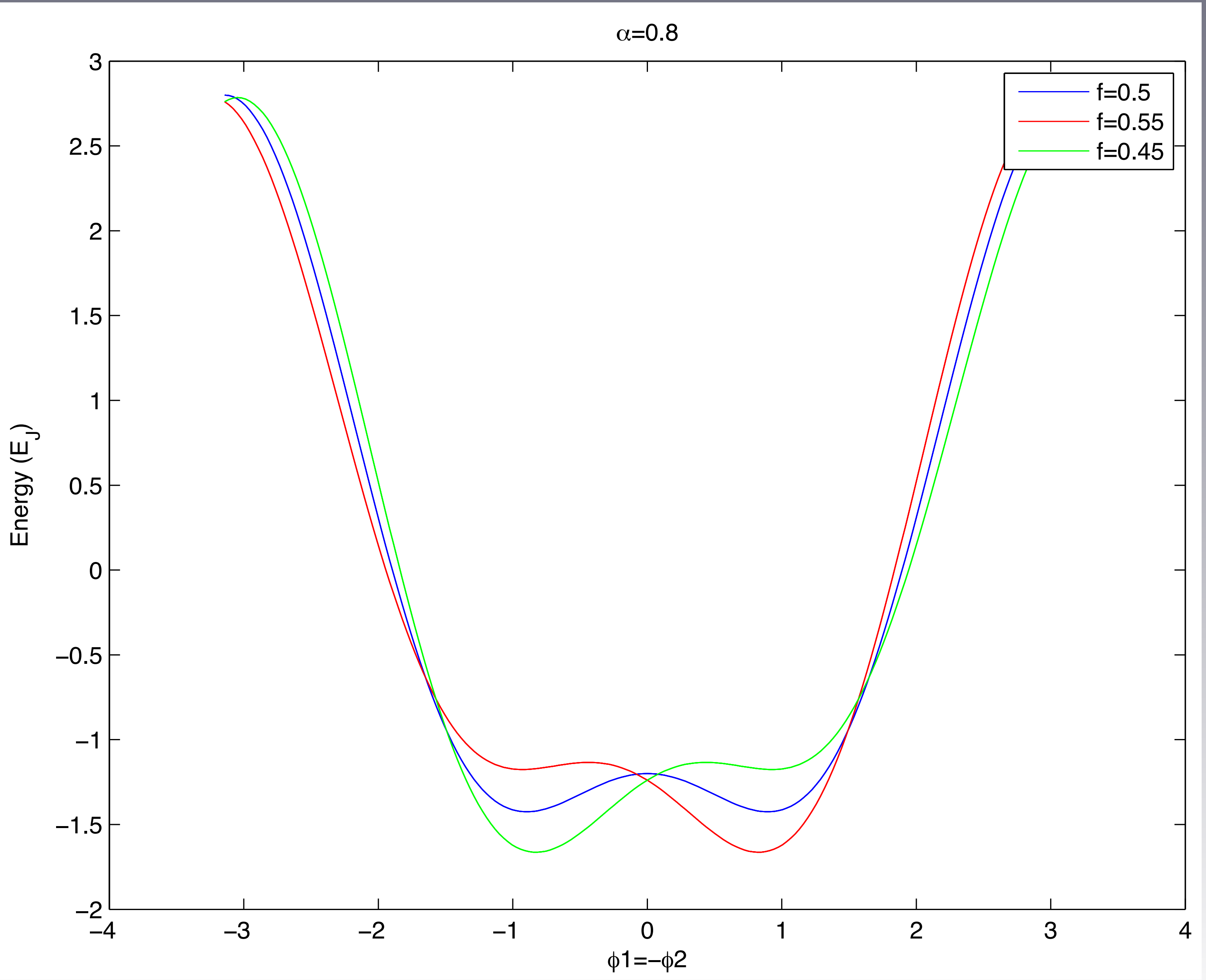
f=0

persistent-current qubit



f=0.5

persistent-current qubit



persistent-current qubit

Classical equations of motion

$$\begin{aligned} I_{\text{left}} &= I_0 \sin(\varphi_1) + C\dot{V} = I_0 \sin(\varphi_1) + C\frac{\phi_0}{2\pi}\ddot{\varphi} \\ I_{\text{right}} &= I_0 \sin(\varphi_2) + C\frac{\phi_0}{2\pi}\ddot{\varphi}_2 \\ I_{\text{top}} &= \alpha I_0 \sin(2\pi f - \varphi_1 + \varphi_2) + \alpha C\frac{\phi_0}{2\pi}(\ddot{\varphi}_2 - \ddot{\varphi}_1) \end{aligned}$$

$$I_{\text{left}} = I_{\text{top}} = -I_{\text{right}}$$

persistent-current qubit

Lagrangian

$$\begin{aligned}\mathcal{L} &= \mathcal{L}(\varphi_1, \varphi_2, \dot{\varphi}_1, \dot{\varphi}_2) \\ &= \frac{C}{2}\left(\frac{\phi_0}{2\pi}\right)^2 \dot{\varphi}_1^2 + \frac{C}{2}\left(\frac{\phi_0}{2\pi}\right)^2 \dot{\varphi}_2^2 + \alpha\frac{C}{2}\left(\frac{\phi_0}{2\pi}\right)^2 (\dot{\varphi}_1 - \dot{\varphi}_2)^2 \\ &\quad + E_J \cos(\varphi_1) + E_J \cos(\varphi_2) + \alpha E_J \cos(\varphi_1 - \varphi_2 - 2\pi f)\end{aligned}$$

The Lagrange equations reproduce the classical eqations.

$$\frac{d}{dt}\left\{\frac{\partial \mathcal{L}}{\partial \dot{\varphi}_i}\right\} - \frac{\partial \mathcal{L}}{\partial \varphi_i} = 0$$

persistent-current qubit

From the Lagrangian one can derive the canonical variables

$$q_1 = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_1} = C\left(\frac{\phi_0}{2\pi}\right)^2 \dot{\varphi}_1 + \alpha C\left(\frac{\phi_0}{2\pi}\right)^2 (\dot{\varphi}_1 - \dot{\varphi}_2)$$

$$q_2 = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_2} = C\left(\frac{\phi_0}{2\pi}\right)^2 \dot{\varphi}_2 - \alpha C\left(\frac{\phi_0}{2\pi}\right)^2 (\dot{\varphi}_1 - \dot{\varphi}_2)$$

$$\dot{\varphi}_1 = \frac{1}{C\left(\frac{\phi_0}{2\pi}\right)^2}\frac{1+\alpha}{1+2\alpha}q_1 + \frac{1}{C\left(\frac{\phi_0}{2\pi}\right)^2}\frac{\alpha}{1+2\alpha}q_2$$

$$\dot{\varphi}_2 = \frac{1}{C\left(\frac{\phi_0}{2\pi}\right)^2}\frac{\alpha}{1+2\alpha}q_1 + \frac{1}{C\left(\frac{\phi_0}{2\pi}\right)^2}\frac{1+\alpha}{1+2\alpha}q_2$$

persistent-current qubit

With a Legendre transformation we get the Hamiltonian

$$\mathcal{H}(q_1, q_2, \varphi_1, \varphi_2) = \dot{\varphi}_1 q_1 + \dot{\varphi}_2 q_2 - \mathcal{L}$$

$$\begin{aligned}\mathcal{H} &= 4\frac{E_J}{\left(2e\frac{\phi_0}{2\pi}\right)^2}\left(\frac{1+\alpha}{1+2\alpha}q_1^2 + \frac{2\alpha}{1+2\alpha}q_1 q_2 + \frac{1+\alpha}{1+2\alpha}q_2^2\right) \\ &\quad -E_J\cos(\varphi_1) - E_J\cos(\varphi_2) - \alpha E_J\cos(\varphi_1 - \varphi_2 - 2\pi f)\end{aligned}$$

From the canonical equations

$$\dot{\varphi}_i = \frac{\partial \mathcal{H}}{\partial q_i}$$

$$\dot{q}_i = -\frac{\partial \mathcal{H}}{\partial \varphi_i}$$

one gets back the classical equations of motion (4).

persistent-current qubit

Qubit Hamiltonian in the Charge Base

using

$$q_1 = 2e\frac{\phi_0}{2\pi}n_1$$
$$q_2 = 2e\frac{\phi_0}{2\pi}n_2$$

$$\mathcal{H} = 4E_c\frac{1+\alpha}{1+2\alpha}n_1^2 + 4E_c\frac{2\alpha}{1+2\alpha}n_1 n_2 + 4E_c\frac{1+\alpha}{1+2\alpha}n_2^2$$
$$-E_J\cos(\varphi_1) - E_J\cos(\varphi_2) - \alpha E_J\cos(\varphi_1 - \varphi_2 - 2\pi f)$$

persistent-current qubit

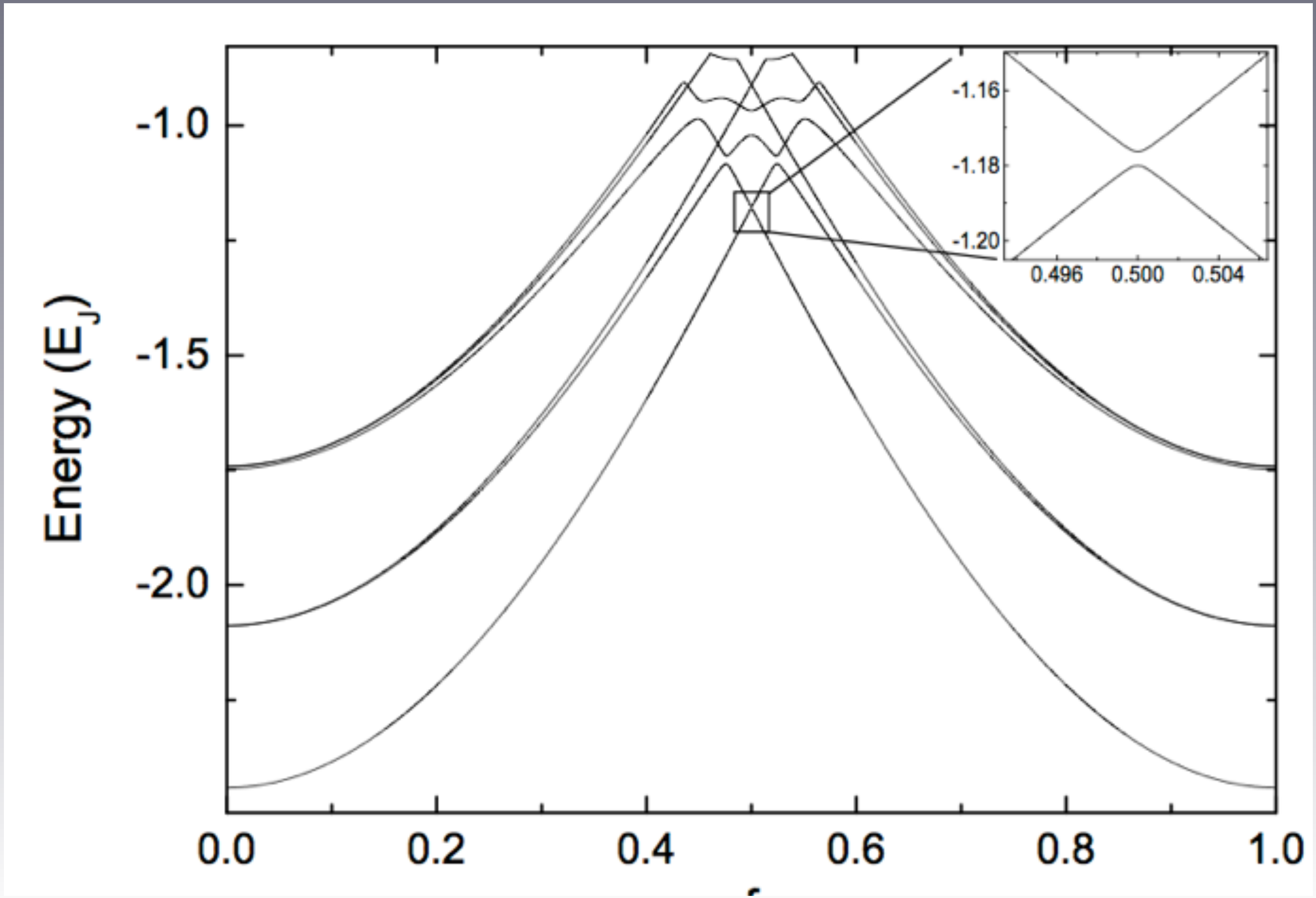
$$\begin{aligned} e^{i\varphi_j}|n_j\rangle &= |n_j+1\rangle \\ e^{-i\varphi_j}|n_j\rangle &= |n_j-1\rangle \\ e^{i(\varphi_j+k)}|n_j\rangle &= e^{ik}|n_j+1\rangle \\ e^{i(\varphi_j-k)}|n_j\rangle &= e^{-ik}|n_j-1\rangle \end{aligned}$$

$$\langle m_1, m_2| - E_J\cos(\varphi_1)|n_1, n_2\rangle = \delta_{m_2,n_2}\frac{-E_J}{2}(\delta_{m_1,n_1-1} + \delta_{m_1,n_1+1})$$

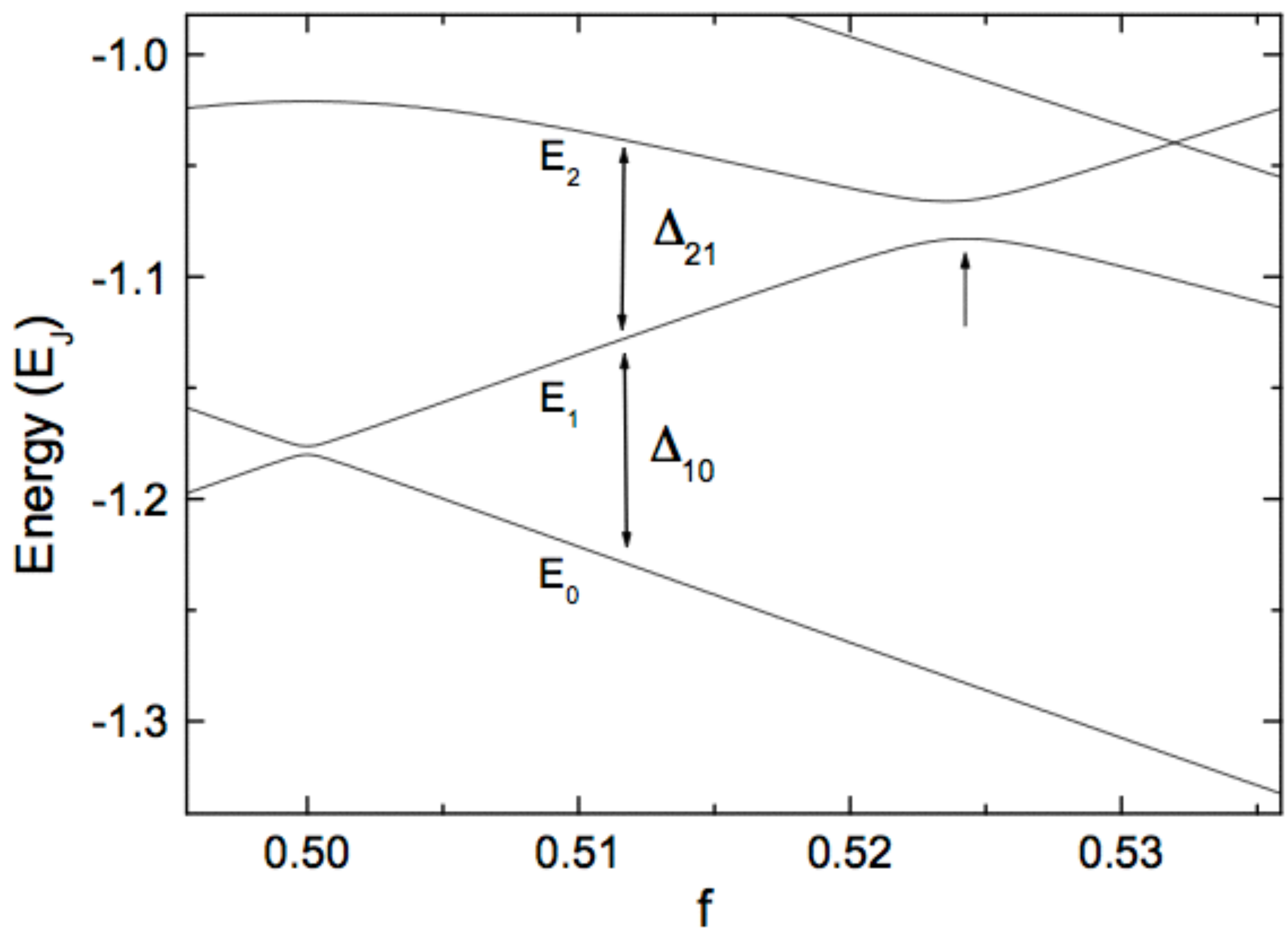
$$\langle m_1, m_2| - E_J\cos(\varphi_2)|n_1, n_2\rangle = \delta_{m_1,n_1}\frac{-E_J}{2}(\delta_{m_2,n_2-1} + \delta_{m_2,n_2+1})$$

$$\langle m_1, m_2| - \alpha E_J\cos(\varphi_1 - \varphi_2 - 2\pi f)|n_1, n_2\rangle =$$
$$\left(\delta_{m_1,n_1+1}\delta_{m_2,n_2-1}e^{-i2\pi f} + \delta_{m_1,n_1-1}\delta_{m_2,n_2+1}e^{i2\pi f}\right)\frac{-\alpha E_J}{2}$$

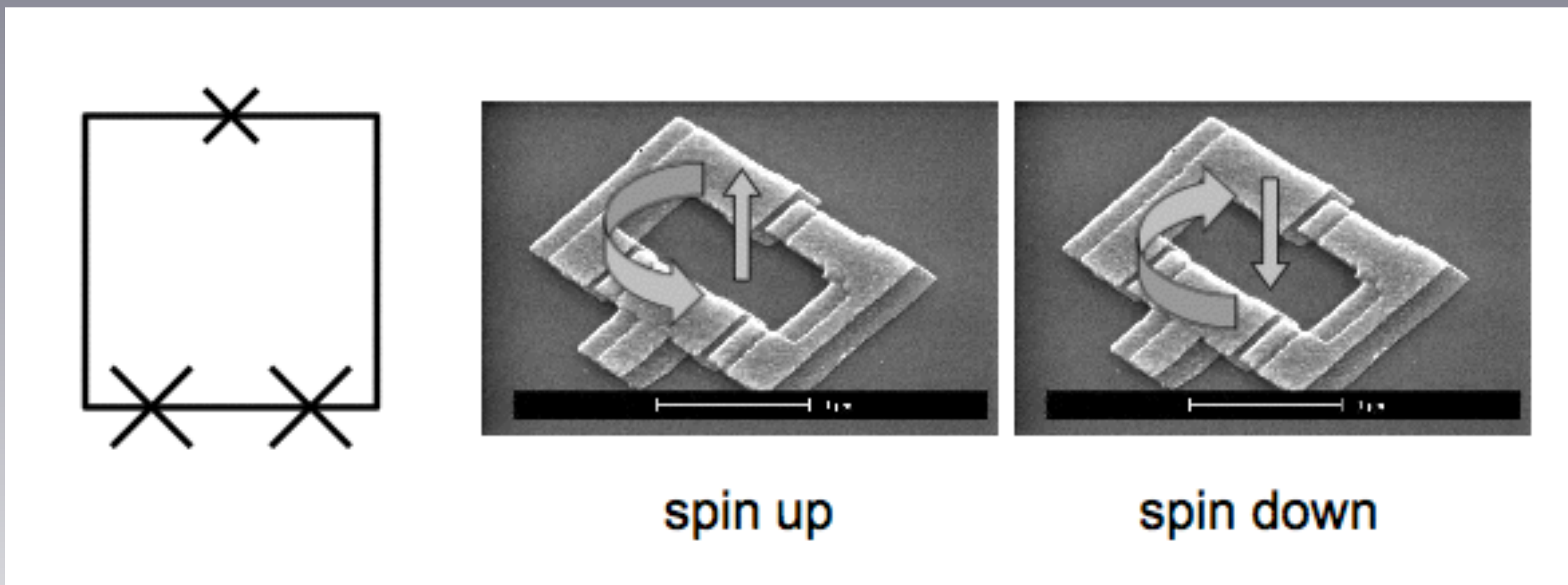
persistent-current qubit



persistent-current qubit



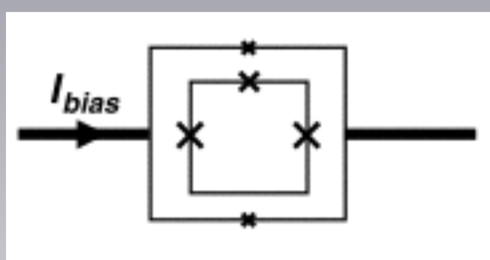
persistent-current qubit

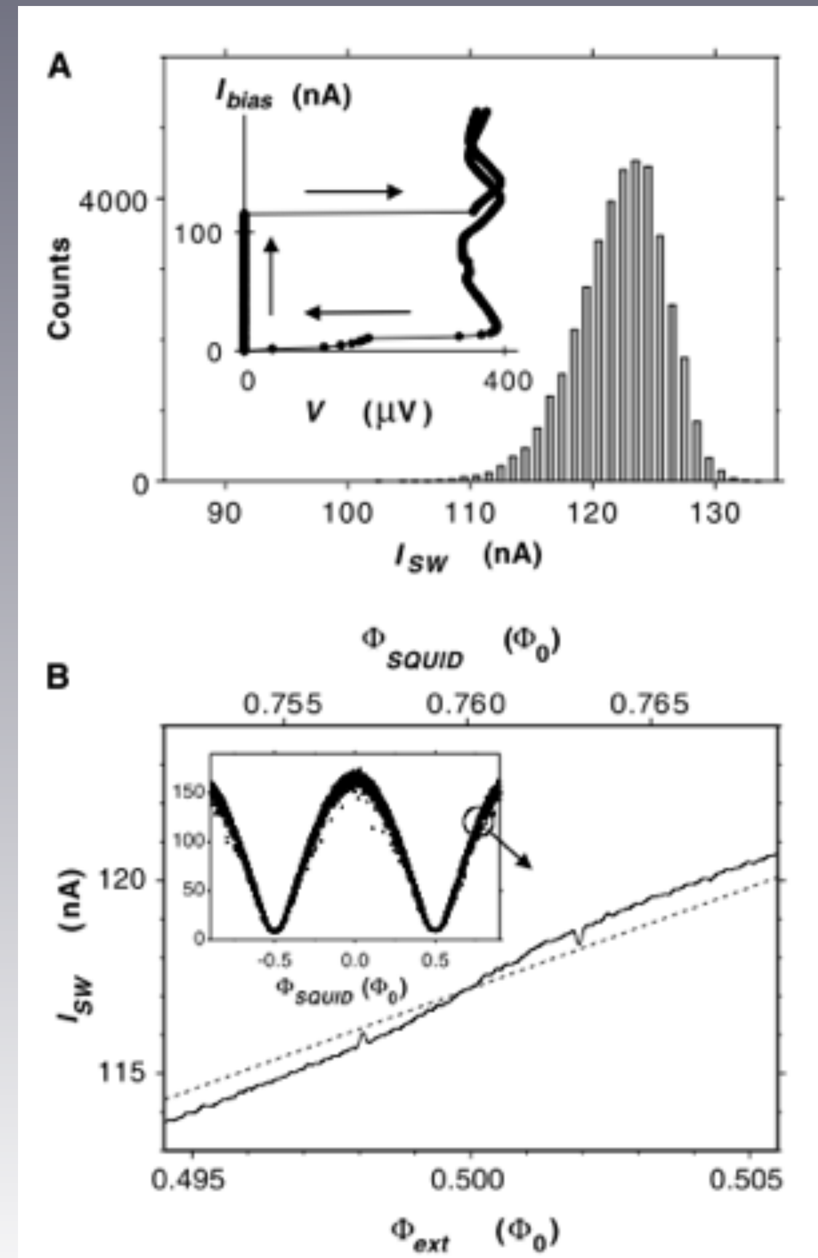


persistent-current qubit

Quantum Superposition of Macroscopic Persistent-Current States

Caspar H. van der Wal,[1*] A. C. J. ter Haar,[1] F. K. Wilhelm,[1] R. N. Schouten,[1] C. J. P. M. Harmans,[1] T. P. Orlando,[2] Seth Lloyd,[3] J. E. Mooij[1,2]





persistent-current qubit